Published by Institute of Physics Publishing for SISSA

Received: *August 26, 2007*
Accepted: *October 9, 2007*
Published: *October 12, 2007*

(p, q)-string in matrix-regularized membrane and type IIB duality

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Abstract: We consider a lightcone wrapped supermembrane compactified on a 2-torus in the matrix regularization. We examine the double dimensional reduction technique and deduce the free matrix string of (p, q)-string in type IIB superstring theory explicitly from the matrix-regularized wrapped supermembrane. In addition we obtain the (2+1)-dimensional super Yang-Mills action in a curved background. We also examine the SL(2) duality in type IIB theory.

Keywords: M-Theory, String Duality, M(atrix) Theories.

JHEP10(2007)053

Contents

1. Introduction

M-theory in eleven dimensions [1, 2] unifies all the five perturbative superstring theories in ten dimensions and it should be reduced to 11-dimensional supergravity theory in its low energy limit. Supermembrane in eleven dimensions [3] is believed to play an important role to understand the dynamics of M-theory. In fact, it was shown that the wrapped supermembrane on $\mathbb{R}^{10} \times S^1$ leads to the type IIA fundamental strings in the shrinking limit of S^1, or by means of the double dimensional reduction [4]. On the other hand, type IIA superstring theory on $\mathbb{R}^9 \times S^1$ is equivalent via T-duality to the type IIB superstring theory on $\mathbb{R}^9 \times \tilde{S}^1$, where $\tilde{S}^1$ is the dual circle whose radius is inversely related to the one of the S^1 [5, 6]. And the shrinking S^1 limit of the type IIA superstring theory on $\mathbb{R}^9 \times S^1$ leads to the type IIB superstring theory in $\mathbb{R}^{10}$. Accordingly, M-theory on $\mathbb{R}^9 \times T^2$ in the shrinking volume limit of T^2 is reduced to the type IIB superstring theory in $\mathbb{R}^{10}$ [7]. It was shown that the type IIB superstring theory contains a bound state of p fundamental strings and q D1-branes (D-strings), which is called a (p, q)-string [8, 9]. It was pointed out that the supermembrane wrapped p times around a compactified direction and q times around the other compactified direction of the target-space is reduced to a (p, q)-string [8]. Recently, type IIB (p,q)-string action was deduced directly from the wrapped supermembrane action on $\mathbb{R}^9 \times T^2$ adopting the double dimensional reduction and T-duality [10], in which the reduced supermembrane is coupled to both the RR and NSNS 2-forms and it has the correct tension of (p,q)-string.

Supermembrane theory is self-interacting and it has continuous energy spectrum [11]. This implies that it is inherently multi-body and has no coupling constant. Thus we cannot directly adopt the ordinary canonical quantization procedure to the supermembrane theory. In order to handle the supermembrane the matrix regularization was introduced as a "quantization" procedure of the supermembrane [12, 13]. Matrix theory [14] is described by $N \times N$ matrices which can be thought of as the spatial component of 10-dimensional super Yang-Mills fields after reducing to 1+0 dimension. This supersymmetric quantum mechanical system is interpreted as the low energy effective theory of D0-branes (D-particles). And it was conjectured that the $N \to \infty$ limit of the system captures all the degrees of freedom of M-theory in the infinite momentum frame.

Matrix theory compactified on S^1 leads to matrix string theory [15 − 17] through the T-duality prescription [18]. And hence matrix string theory can be thought of as (1+1)-dimensional super Yang-Mills theory describing the low energy effective theory of D-strings. It is also conjectured to give a non-perturbative definition of the type IIA superstring theory. Similarly, the matrix regularization of wrapped supermembrane on $\mathbb{R}^9 \times S^1$ leads to matrix string theory [19 − 21]. Furthermore, the matrix regularization procedure of wrapped supermembrane on $\mathbb{R}^9 \times T^2$ was introduced [22] and it was shown that the regularized theory is T-dual to (2+1)-dimensional super Yang-Mills theory [22] which is low energy effective theory of D2-branes.

The purpose of this paper is to deduce matrix (p,q)-strings directly from a matrix-regularized lightcone supermembrane compactified on a 2-torus referring to the analysis in ref. [10]. In addition, following the lines of [22], we will obtain the (2+1)-dimensional super Yang-Mills theory in a curved background from the matrix regularized wrapped supermembrane and we examine the duality in the dimensionally reduced type II string. We should note that the curved background fields are not mapped to matrix-valued background fields, or they are proportional to the unit matrix in the matrix regularization. The background plays the role of probing the membrane or (p,q)-string. That is, we shall see that both the NSNS and RR 2-forms are coupled to the matrix regularized (p,q)-string.

The plan of this paper is as follows. In section 2 we mainly review the matrix regularization of the lightcone wrapped supermembrane on $\mathbb{R}^9 \times T^2$ [22] to fix the notations, which are used in the following sections. In section 3 we will consider a lightcone wrapped supermembrane compactified on a 2-torus in a curved background and apply matrix regularization technique to it. Then we adopt the double dimensional reduction technique and derive the Green-Schwarz (p,q)-string. In section 4 we also start with the wrapped supermembrane. Then we apply the matrix regularization technique with a suitable choice of the matrix representation to give a standard form of super Yang-Mills action in a curved background and we consider the SL$(2,\mathbb{R})$ transformation and type IIB string duality. The section 5 is devoted to summary and discussion.

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2. Matrix-regularized wrapped supermembrane in flat background

The 11-dimensional supermembrane in the lightcone gauge[1] is given by (only bosonic degrees of freedom are presented here)

$$S = \frac{LT}{2}\int d\tau \int_0^{2\pi} d\sigma^1 d\sigma^2 \left[(D_\tau X^M)^2 - \frac{1}{2L^2}\{\, X^M, X^N \,\}^2\right], \tag{2.1}$$

$$D_\tau X^M = \partial_\tau X^M - \frac{1}{L}\{\, A, X^M \,\}, \tag{2.2}$$

$$\{\, A, B \,\} \equiv \epsilon^{ij}\partial_{\sigma^i} A \partial_{\sigma^j} B\,, \tag{2.3}$$

where $\partial_\tau = \partial/\partial\tau$, $\partial_{\sigma^i} = \partial/\partial\sigma^i$, $i, j = 1, 2$, $\epsilon^{12} = -\epsilon^{21} = 1, \epsilon^{11} = \epsilon^{22} = 0$, $M, N = 1, 2, \cdots, 9$, X^M is the target-space coordinates and A is the gauge field, T is the tension of the supermembrane and L is an arbitrary parameter of mass dimension -1.[2] This theory has the area preserving diffeomorphisms (APD) of the spacesheet as a residual symmetry. Note that L can be changed for L' by a simple rescaling of $\tau \to (L/L')\,\tau$.

Let us consider the wrapped supermembrane on $\mathbb{R}^9 \times T^2$ taking X^8 and X^9 as the coordinates of the two cycles of the T^2. Then the target-space coordinates X^M and the gauge A are expanded as[3]

$$X^9(\sigma^1, \sigma^2) = w_1 L_1 \sigma^2 + \sum_{k_1,k_2=-\infty}^{\infty} Y^1_{(k_1,k_2)}\, e^{ik_1\sigma^1 + ik_2\sigma^2} = w_1 L_1 \sigma^2 + Y^1(\sigma^1, \sigma^2), \tag{2.4}$$

$$X^8(\sigma^1, \sigma^2) = w_2 L_2 \sigma^1 + \sum_{k_1,k_2=-\infty}^{\infty} Y^2_{(k_1,k_2)}\, e^{ik_1\sigma^1 + ik_2\sigma^2} \equiv w_2 L_2 \sigma^1 + Y^2(\sigma^1, \sigma^2), \tag{2.5}$$

$$X^m(\sigma^1, \sigma^2) = \sum_{k_1,k_2=-\infty}^{\infty} X^m_{(k_1,k_2)}\, e^{ik_1\sigma^1 + ik_2\sigma^2}, \tag{2.6}$$

$$A(\sigma^1, \sigma^2) = \sum_{k_1,k_2=-\infty}^{\infty} A_{(k_1,k_2)}\, e^{ik_1\sigma^1 + ik_2\sigma^2}, \tag{2.7}$$

where $m = 1, 2, \cdots, 7$, L_1 and L_2 are the radii of the two cycles of T^2 and $w_1, w_2\,(\neq 0)$ are integers. These fields satisfy the periodicity conditions,

$$X^9(\sigma^1, \sigma^2 + 2\pi) = 2\pi w_1 L_1 + X^9(\sigma^1, \sigma^2)\,, \tag{2.8}$$

$$X^8(\sigma^1, \sigma^2 + 2\pi) = X^8(\sigma^1, \sigma^2)\,, \tag{2.9}$$

$$X^9(\sigma^1 + 2\pi, \sigma^2) = X^9(\sigma^1, \sigma^2)\,, \tag{2.10}$$

$$X^8(\sigma^1 + 2\pi, \sigma^2) = 2\pi w_2 L_2 + X^8(\sigma^1, \sigma^2)\,, \tag{2.11}$$

$$X^m(\sigma^1 + 2\pi, \sigma^2) = X^m(\sigma^1, \sigma^2 + 2\pi) = X^m(\sigma^1, \sigma^2)\,, \tag{2.12}$$

$$A(\sigma^1 + 2\pi, \sigma^2) = A(\sigma^1, \sigma^2 + 2\pi) = A(\sigma^1, \sigma^2)\,. \tag{2.13}$$

[1]In this paper, we consider only toroidal membranes. Precisely speaking, in this case, we need to impose the global constraints associated with the information of the global topology [23].

[2]The mass dimensions of the world-volume parameters, τ, σ^1 and σ^2, are 0.

[3]The τ-dependence is not written explicitly for all the variables.

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These represent the supermembrane wrapping w_1-times around one of the two compact directions X^9 and w_2-times around the other direction X^8. We call these two cycles $(0,1)$- and $(1,0)$-cycles, respectively. Plugging eqs. (2.4)–(2.7) into the covariant derivatives and the Poisson brackets, we have

$$
\begin{aligned}
F_{\tau\sigma^1} &\equiv D_\tau X^9 = \partial_\tau Y^1 - \frac{w_1 L_1}{L}\partial_{\sigma^1} A - \frac{1}{L}\{\, A, Y^1 \,\}\,, && (2.14)\\
F_{\tau\sigma^2} &\equiv D_\tau X^8 = \partial_\tau Y^2 + \frac{w_2 L_2}{L}\partial_{\sigma^2} A - \frac{1}{L}\{\, A, Y^2 \,\}\,, && (2.15)\\
F_{\sigma^1\sigma^2} &\equiv \frac{1}{L}\{\, X^8, X^9 \,\} = \frac{w_1 w_2 L_1 L_2}{L} + \frac{w_1 L_1}{L}\,\partial_{\sigma^1} Y^2 \\
&\qquad + \frac{w_2 L_2}{L}\,\partial_{\sigma^2} Y^1 + \frac{1}{L}\{\, Y^2, Y^1 \,\}\,, && (2.16)\\
D_\tau X^m &= \partial_\tau X^m - \frac{1}{L}\{\, A, X^m \,\}\,, && (2.17)\\
D_{\sigma^1} X^m &\equiv \frac{1}{L}\{\, X^9, X^m \,\} = -\frac{w_1 L_1}{L}\,\partial_{\sigma^1} X^m + \frac{1}{L}\{\, Y^1, X^m \,\}\,, && (2.18)\\
D_{\sigma^2} X^m &\equiv \frac{1}{L}\{\, X^8, X^m \,\} = \frac{w_2 L_2}{L}\,\partial_{\sigma^2} X^m + \frac{1}{L}\{\, Y^2, X^m \,\}\,. && (2.19)
\end{aligned}
$$

Thus, the action (2.1) is rewritten by

$$
\begin{aligned}
S = \frac{LT}{2}\int d\tau \int_0^{2\pi} d\sigma^1 d\sigma^2 \Big[& F_{\tau\sigma^1}^2 + F_{\tau\sigma^2}^2 - F_{\sigma^1\sigma^2}^2 \\
& + (D_\tau X^m)^2 - (D_{\sigma^1} X^m)^2 - (D_{\sigma^2} X^m)^2 - \frac{1}{2L^2}\{\, X^m, X^n \,\}^2 \Big]\,. \qquad (2.20)
\end{aligned}
$$

2.1 The matrix representation

Here we shall consider the matrix regularization of the wrapped supermembrane on $\mathbb{R}^9 \times T^2$, eqs. (2.1)–(2.7). The procedure for the matrix regularization is the following [22]: (i) Introduce the noncommutativity on the spacesheet of supermembrane, or replace the product of functions on the spacesheet to the star-product. (ii) If possible, find the central elements of the star-commutator algebra and truncate the generators of the algebra consistently. (iii) Give a matrix representation of the (truncated) star-commutator algebra.

The star-commutators algebra for the set of generators $\{e^{ik_1\sigma^1 + ik_2\sigma^2}, \sigma^1, \sigma^2 \mid k_1, k_2 \in \mathbb{Z}\}$ is given by [20 − 22, 24]

$$
\begin{aligned}
[\, e^{ik_1\sigma^1 + ik_2\sigma^2}, e^{ik_1'\sigma^1 + ik_2'\sigma^2} \,]_* &= -2i \sin\left(\frac{\pi}{N}\,\epsilon^{ij} k_i k_j'\right)\, e^{i(k_1+k_1')\sigma^1 + i(k_2+k_2')\sigma^2}\,, && (2.21)\\
[\, \sigma^1, e^{ik_1\sigma^1 + ik_2\sigma^2} \,]_* &= -\frac{2\pi k_2}{N}\, e^{ik_1\sigma^1 + ik_2\sigma^2}\,, && (2.22)\\
[\, \sigma^2, e^{ik_1\sigma^1 + ik_2\sigma^2} \,]_* &= \frac{2\pi k_1}{N}\, e^{ik_1\sigma^1 + ik_2\sigma^2}\,, && (2.23)\\
[\, \sigma^1, \sigma^2 \,]_* &= i\frac{2\pi}{N}\,. && (2.24)
\end{aligned}
$$

Since we can not find a central element of the algebra eqs. (2.21)–(2.24), the truncation is not possible in T^2 compactified case. The generators are represented by $N\times N$ matrices with two continuous parameters θ^1,θ^2 [22],[4]

$$e^{i(u_1N+v_1)\sigma^1+i(u_2N+v_2)\sigma^2} \rightarrow e^{i(u_1N+v_1)\theta^1/N}e^{-i(u_2N+v_2)\theta^2/N}\lambda^{-v_1v_2/2}\,V^{v_2}\,U^{v_1}, \quad (2.25)$$

$$\sigma^2 \rightarrow -2\pi i\partial_{\theta^1}I_N, \quad (2.26)$$

$$\sigma^1 \rightarrow -2\pi i\partial_{\theta^2}I_N+\frac{\theta^1}{N}I_N\,, \quad (2.27)$$

where $\lambda=e^{i2\pi/N}$, $u_1,u_2\in\mathbb{Z}$, $v_1,v_2=0,\pm1,\pm2,\cdots,\pm M$,[5] I_N is the $N\times N$ unit matrix and U,V are the $N\times N$ clock and shift matrices, respectively,

$$U=\begin{pmatrix}1 & & & & 0\\ & \lambda & & & \\ & & \lambda^2 & & \\ & & & \ddots & \\ 0 & & & & \lambda^{N-1}\end{pmatrix},\qquad V=\begin{pmatrix}0 & 1 & & & \\ & 0 & 1 & & \\ \vdots & & \ddots & \ddots & \\ 0 & & & 0 & 1\\ 1 & 0 & \cdots & & 0\end{pmatrix}. \quad (2.28)$$

These have the following properties,

$$U^N=V^N=I_N\,,\quad VU=\lambda\,UV\,. \quad (2.29)$$

Then, the functions X^9,X^8,X^m and A of σ^1 and σ^2,[6] or eqs. (2.4)–(2.7) are represented by the $N\times N$ matrices

$$X^9(\sigma^1,\sigma^2) \rightarrow -2\pi iw_1L_1\partial_{\theta^1}I_N+Y^1(\theta^1,\theta^2)\,, \quad (2.30)$$

$$X^8(\sigma^1,\sigma^2) \rightarrow -2\pi iw_2L_2\partial_{\theta^2}I_N+\frac{w_2L_2}{N}\theta^1I_N+Y^2(\theta^1,\theta^2), \quad (2.31)$$

$$X^m(\sigma^1,\sigma^2) \rightarrow X^m(\theta^1,\theta^2), \quad (2.32)$$

$$A(\sigma^1,\sigma^2) \rightarrow A(\theta^1,\theta^2), \quad (2.33)$$

where (Ξ represents Y^1, Y^2, X^m and A)

$$\Xi(\theta^1,\theta^2)=\sum_{u_1,u_2\in\mathbb{Z}}\sum_{v_1,v_2=-M}^{M}\Xi_{(u_1N+v_1,u_2N+v_2)}e^{i(u_1N+v_1)\theta_1/N}e^{-i(u_2N+v_2)\theta_2/N}\lambda^{-v_1v_2/2}V^{v_2}U^{v_1}. \quad (2.34)$$

Note that $\Xi(\theta^1,\theta^2)$ satisfies the boundary condition [22],

$$\Xi(\theta^1+2\pi,\theta^2)=V\,\Xi(\theta^1,\theta^2)\,V^\dagger,\quad \Xi(\theta^1,\theta^2+2\pi)=U\,\Xi(\theta^1,\theta^2)\,U^\dagger. \quad (2.35)$$

[4] Note that the parameters θ^i are, in principle, independent of the spacesheet coordinates σ^1,σ^2.

[5] We assume N odd, $N=2M+1$ and we have parametrized k_i as $k_i=u_iN+v_i$ $(i=1,2)$.

[6] Of course they are also functions of τ. We just do not mention it explicitly.

Furthermore, the Poisson bracket and the double integrals are represented as follows,

$$\{\,\cdot\,,\,\cdot\,\} \rightarrow -i\frac{N}{2\pi}\,[\,\cdot\,,\,\cdot\,]\,, \tag{2.36}$$

$$\int_0^{2\pi} d\sigma^1 d\sigma^2 \,* \rightarrow \frac{1}{N}\int_0^{2\pi} d\theta^1 d\theta^2\,\mathrm{Tr}[\,*\,]\,. \tag{2.37}$$

Thus, from these results with a rescaling $\tau \rightarrow \tau/N$, the matrix-regularized action of the wrapped membrane on $\mathbb{R}^9 \times T^2$ is given by

$$\begin{aligned} S_{2+1} = \frac{LT}{2}\int d\tau \int_0^{2\pi} d\theta^1 d\theta^2\,\mathrm{Tr}\Big[&(F_{\tau\theta^1})^2 + (F_{\tau\theta^2})^2 - (F_{\theta^1\theta^2})^2 \\ &+(D_\tau X^m)^2 - (D_{\theta^1}X^m)^2 - (D_{\theta^2}X^m)^2 + \frac{1}{2(2\pi L)^2}\,[\,X^m, X^n\,]^2\Big]\,, \end{aligned} \tag{2.38}$$

where

$$F_{\tau\theta^1} = \partial_\tau Y^1 - \frac{w_1L_1}{L}\partial_{\theta^1}A + \frac{i}{2\pi L}[\,A, Y^1\,]\,, \tag{2.39}$$

$$F_{\tau\theta^2} = \partial_\tau Y^2 - \frac{w_2L_2}{L}\partial_{\theta^2}A + \frac{i}{2\pi L}[\,A, Y^2\,]\,, \tag{2.40}$$

$$F_{\theta^1\theta^2} = \frac{w_1w_2L_1L_2}{NL}\,I_N + \frac{w_1L_1}{L}\,\partial_{\theta^1}Y^2 - \frac{w_2L_2}{L}\,\partial_{\theta^2}Y^1 + \frac{i}{2\pi L}\,[\,Y^1, Y^2\,]\,, \tag{2.41}$$

$$D_\tau X^m = \partial_\tau X^m + \frac{i}{2\pi L}\,[\,A, X^m\,]\,, \tag{2.42}$$

$$D_{\theta^1}X^m = \frac{w_1L_1}{L}\,\partial_{\theta^1}X^m + \frac{i}{2\pi L}\,[\,Y^1, X^m\,]\,, \tag{2.43}$$

$$D_{\theta^2}X^m = \frac{w_2L_2}{L}\,\partial_{\theta^2}X^m + \frac{i}{2\pi L}\,[\,Y^2, X^m\,]\,. \tag{2.44}$$

Note that the fields Y^1, Y^2, X^m and A have mass dimension -1 and the parameters τ, θ^1, θ^2 have mass dimension 0. We also rewrite the action (2.38) to the standard form of Yang-Mills theory. In order to adjust the mass dimensions of the fields and the parameters, we rewrite them by introducing some dimensionful constants,

$$Y^1(\theta^1,\theta^2) \rightarrow \alpha A_1(x^1,x^2)\,, \tag{2.45}$$

$$Y^2(\theta^1,\theta^2) \rightarrow \alpha A_2(x^1,x^2)\,, \tag{2.46}$$

$$X^m(\theta^1,\theta^2) \rightarrow \alpha \phi^m(x^1,x^2)\,, \tag{2.47}$$

$$A(\theta^1,\theta^2) \rightarrow \alpha A_0(x^1,x^2)\,, \tag{2.48}$$

$$\theta^1 \rightarrow x^1/\Sigma_1\,, \tag{2.49}$$

$$\theta^2 \rightarrow x^2/\Sigma_2\,, \tag{2.50}$$

$$\tau \rightarrow x^0/\Sigma\,, \tag{2.51}$$

where α has mass dimension -2 and Σ_1, Σ_2 and Σ have mass dimension -1. Then, the

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action (2.38) is rewritten by

$$S_{2+1} = \frac{LT}{2}\frac{1}{\Sigma_1\Sigma_2\Sigma}\int dx^0\int_0^{2\pi\Sigma_1}dx^1\int_0^{2\pi\Sigma_2}dx^2\ \mathrm{Tr}\Big[(F_{\tau\theta^1})^2+(F_{\tau\theta^2})^2-(F_{\theta^1\theta^2})^2 + (D_\tau X^m)^2-(D_{\theta^1}X^m)^2-(D_{\theta^2}X^m)^2+\frac{\alpha^4}{2(2\pi L)^2}[\,\phi^m,\phi^n\,]^2\Big], \tag{2.52}$$

$$F_{\tau\theta^1} = \Sigma\alpha\partial_0 A_1 - \frac{L_1}{L}\Sigma_1\alpha\partial_1 A_0 + i\frac{\alpha^2}{2\pi L}[\,A_0, A_1\,], \tag{2.53}$$

$$F_{\tau\theta^2} = \Sigma\alpha\partial_0 A_2 - \frac{L_2}{L}\Sigma_2\alpha\partial_2 A_0 + i\frac{\alpha^2}{2\pi L}[\,A_0, A_2\,], \tag{2.54}$$

$$F_{\theta^1\theta^2} = \frac{L_1L_2}{NL}I_N + \frac{L_1}{L}\Sigma_1\alpha\partial_1 A_2 - \frac{L_2}{L}\Sigma_2\alpha\partial_2 A_1 + i\frac{\alpha^2}{2\pi L}[\,A_1, A_2\,], \tag{2.55}$$

$$D_\tau X^m = \Sigma\alpha\partial_0\phi^m + i\frac{\alpha^2}{2\pi L}[\,A_0,\phi^m\,], \tag{2.56}$$

$$D_{\theta^1}X^m = \frac{L_1}{L}\Sigma_1\alpha\partial_1\phi^m + i\frac{\alpha^2}{2\pi L}[\,A_1,\phi^m\,], \tag{2.57}$$

$$D_{\theta^2}X^m = \frac{L_2}{L}\Sigma_2\alpha\partial_2\phi^m + i\frac{\alpha^2}{2\pi L}[\,A_2,\phi^m\,], \tag{2.58}$$

where $\partial_0 \equiv \partial/\partial x^0$ and $\partial_i \equiv \partial/\partial x^i$. Here we have put $w_1 = w_2 = 1$ for simplicity. In order to bring the field strength (2.53)–(2.55) into the standard form, we obtain the following relations [22] [7]

$$\Sigma = \frac{\alpha}{2\pi L}, \tag{2.59}$$

$$\Sigma_1 = \frac{\alpha}{2\pi L_1}, \tag{2.60}$$

$$\Sigma_2 = \frac{\alpha}{2\pi L_2}. \tag{2.61}$$

Then, we have obtained the standard form of a bosonic part of (2+1)-dimensional maximally supersymmetric U(N) Yang-Mills theory with the constant magnetic flux,

$$S_{2+1} = \frac{1}{2g_{YM}^2}\int dx^0\int_0^{2\pi\Sigma_1}dx^1\int_0^{2\pi\Sigma_2}dx^2\ \mathrm{Tr}\Big[(F_{01})^2+(F_{02})^2-(F_{12})^2 + (D_0\phi^m)^2-(D_1\phi^m)^2-(D_2\phi^m)^2+\frac{1}{2}[\,\phi^m,\phi^n\,]^2\Big], \tag{2.62}$$

$$F_{01} = \partial_0 A_1 - \partial_1 A_0 + i[\,A_0, A_1\,], \tag{2.63}$$

$$F_{02} = \partial_0 A_2 - \partial_2 A_0 + i[\,A_0, A_2\,], \tag{2.64}$$

$$F_{12} = f_{12} + \partial_1 A_2 - \partial_2 A_1 + i[\,A_1, A_2\,], \tag{2.65}$$

$$D_\alpha\phi^m = \partial_\alpha\phi^m + i[\,A_\alpha,\phi^m\,], \qquad (\alpha = 0,1,2) \tag{2.66}$$

with the boundary conditions (Ξ stands for A_α and ϕ^m),

$$\Xi(x^1+2\pi\Sigma_1, x^2) = V\Xi(x^1,x^2)V^\dagger, \tag{2.67}$$

$$\Xi(x^1, x^2+2\pi\Sigma_2) = U\Xi(x^1,x^2)U^\dagger, \tag{2.68}$$

[7] Eqs. (2.60) and (2.61) represent the T-duality which relates the radii (L_1, L_2) of the 2-torus in M-theory and the those (Σ_1, Σ_2) of the dual 2-torus in the super Yang-Mills theory. We should stress that we have obtained the same relations from the different viewpoint [22].

where the constant magnetic flux f_{12} is given by

$$f_{12} = \frac{1}{2\pi N \Sigma_1 \Sigma_2} I_N \,, \tag{2.69}$$

and g_{YM} is the gauge coupling constant of mass dimension one half, which is given by

$$g_{YM}^2 = (2\pi)^{-2} (\Sigma_1 \Sigma_2)^{-1/2} \, (L_1 L_2)^{-3/2} \, T^{-1}. \tag{2.70}$$

We also define the dimensionless gauge coupling constant $\bar{g}_{YM}$ by

$$\bar{g}_{YM}^2 \equiv g_{YM}^2 (2\pi\Sigma_1 2\pi\Sigma_2)^{1/2} = (2\pi)^{-1} (L_1 L_2)^{-3/2} \, T^{-1} = \frac{2\pi l_{11}^3}{(L_1 L_2)^{3/2}} \,, \tag{2.71}$$

where l_{11} is the 11-dimensional Planck length related to T by $T^{-1} = (2\pi)^2 l_{11}^3$. This dimensionless gauge coupling constant exactly agrees with that obtained in ref. [26] including the numerical constant.[8] Note that in refs. [26] the super Yang-Mills theory was regarded as the low energy effective theory of D-branes in deriving such a relation, while we have taken a different approach of matrix regularization of supermembrane in this section. Furthermore, the constant magnetic flux f_{12} in eq. (2.65) agrees with that obtained in refs. [25, 26] including the numerical constant.

2.2 The general matrix representation

We have adopted a simple representation (2.25)–(2.27) for the star-commutator algebra (2.21)–(2.24) to bring the wrapped supermembrane action to the standard form of super Yang-Mills theory in eqs. (2.62)–(2.66). However, we could adopt more general representation of the algebra

$$\begin{aligned} e^{ik_1\sigma^1 + ik_2\sigma^2} &\to e^{ik_i T^i{}_j \, \theta^j/N} \lambda^{-v_1 v_2/2} \, V^{v_2} \, U^{v_1}, &(2.72)\\ \sigma^2 &\to c^i \partial_{\theta^i} I_N + d_i \theta^i I_N, &(2.73)\\ \sigma^1 &\to e^i \partial_{\theta^i} I_N + f_i \theta^i I_N \,. &(2.74) \end{aligned}$$

In fact, we can easily check that this is also a representation of the star-commutator algebra with following constraints

$$\begin{aligned} ik_i \, T^i{}_j \, e^j &= -2\pi k_2 \,, &(2.75)\\ ik_i \, T^i{}_j \, c^j &= 2\pi k_1 \,, &(2.76)\\ e^i d_i - c^i f_i &= \frac{2\pi i}{N} \,, &(2.77) \end{aligned}$$

where the matrix $T^i{}_j$ is given by

$$T^i{}_j = \frac{2\pi i}{(c^1 e^2 - c^2 e^1)} \begin{pmatrix} -e^2 & e^1 \\ -c^2 & c^1 \end{pmatrix} . \tag{2.78}$$

Note that in such general representation the resultant action is not always in the standard form of the super Yang-Mills action. In section 4 we shall consider the supermembrane wrapped around the general two-cycles of T^2. Then we shall use this general representation to bring the wrapped supermembrane action into the standard super Yang-Mills action.

[8] Note that the parameters Σ_1, Σ_2 and L_1, L_2 in ref. [26] represent the circumferences but not the radii.

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3. Wrapped supermembrane in curved background

In this section we consider the supermembrane wrapped around nontrivial two cycles of T^2 and apply matrix regularization procedure to it. Then we perform the double dimensional reduction and derive the matrix (p,q)-strings.

3.1 Setup

The bosonic part of the lightcone supermembrane in 11-dimensional curved background is given in ref. [27, 28]. It was conjectured in ref. [29] to identify a lightcone component of the background 3-form A_{-MN} with the noncommutative parameter of the 2-torus. We need more study on this issue, however, since our goal in this section is to deduce the Green-Schwarz (p,q)-string action from the matrix-regularized wrapped supermembrane by the double dimensional reduction, we shall put the background fields along the lightcone directions zero. Then the action in ref. [27] is reduced to contain only fields with the transverse indices,

$$\begin{aligned} S = \frac{LT}{2}\int d\tau \int_0^{2\pi} d\sigma^1 d\sigma^2 \Big[&(D_\tau X^M)^2 \\ &-\frac{1}{2L^2}\{X^M, X^N\}^2 + \frac{1}{L} D_\tau X^M A_{MNP}\{X^N, X^P\}\Big], \end{aligned} \tag{3.1}$$

where A_{MNP} is the 3-form field, X^M is target-space coordinates and the transverse indices $M,N,P = 1,2,\cdots,9$ are contracted by the target-space metric G_{MN}. Considering the line element on a 2-torus

$$ds^2_{T^2} = G_{uv}\, dX^u dX^v = \left(G_{88} - \frac{(G_{89})^2}{G_{99}}\right)(dX^8)^2 + G_{99}\left(dX^9 + \frac{G_{89}}{G_{99}} dX^8\right)^2, \tag{3.2}$$

where $u,v = 8,9$, we shall choose the target-space coordinates satisfying the following boundary conditions [10]

$$\sqrt{G_{99}}\, X^9(\sigma^1, \sigma^2 + 2\pi) = 2\pi w_1 L_1 p + \sqrt{G_{99}}\, X^9(\sigma^1,\sigma^2), \tag{3.3}$$

$$\sqrt{G_{88} - \frac{(G_{89})^2}{G_{99}}}\, X^8(\sigma^1, \sigma^2 + 2\pi) = 2\pi w_1 L_2 q + \sqrt{G_{88} - \frac{(G_{89})^2}{G_{99}}}\, X^8(\sigma^1,\sigma^2), \tag{3.4}$$

$$\sqrt{G_{99}}\, X^9(\sigma^1 + 2\pi, \sigma^2) = 2\pi w_2 L_1 r + \sqrt{G_{99}}\, X^9(\sigma^1,\sigma^2), \tag{3.5}$$

$$\sqrt{G_{88} - \frac{(G_{89})^2}{G_{99}}}\, X^8(\sigma^1 + 2\pi, \sigma^2) = 2\pi w_2 L_2 s + \sqrt{G_{88} - \frac{(G_{89})^2}{G_{99}}}\, X^8(\sigma^1,\sigma^2). \tag{3.6}$$

or

$$X^9(\sigma^1,\sigma^2) = R_1\,(w_1 p \sigma^2 + w_2 r \sigma^1) + Y^1(\sigma^1,\sigma^2), \tag{3.7}$$

$$X^8(\sigma^1,\sigma^2) = R_2\,(w_1 q \sigma^2 + w_2 s \sigma^1) + Y^2(\sigma^1,\sigma^2), \tag{3.8}$$

where[9]

$$pr + qs = 0, \quad ps - qr \equiv n_c > 0, \quad (p,q,r,s \in \mathbb{Z},\ w_1 \in \mathbb{N}\backslash\{0\},\ w_2 \in \mathbb{Z}\backslash\{0\}) \tag{3.9}$$

[9] We may assume $n_c > 0$ and $w_1 > 0$ without loss of generality since we can flip the signs of $(p,q) \to (-p,-q)$ (for w_1) and $(r,s) \to (-r,-s)$ (for n_c) if necessary. Furthermore, we may see that eq. (3.9) leads to $(r,s) = n(-q,p)$ $(n \in \mathbb{N})$.

and[10]

$$R_1 \equiv \frac{L_1}{\sqrt{G_{99}}}\,, \quad R_2 \equiv \frac{L_2}{\sqrt{G_{88} - \frac{(G_{89})^2}{G_{99}}}}\,. \tag{3.10}$$

Y^i ($i = 1, 2$) and the other fields in eqs. (2.6) and (2.7) satisfy the periodic boundary conditions, $Y^1(\sigma^1 + 2\pi, \sigma^2) = Y^1(\sigma^1, \sigma^2 + 2\pi) = Y^1(\sigma^1, \sigma^2)$, etc.. The above expressions represent that the supermembrane is wrapping $w_1 p$-times around one of the two compact directions (X^9) and $w_1 q$-times around the other direction (X^8), or w_1-times around (p, q)-cycle along the σ^2-direction on the worldsheet. And also it is wrapping w_2-times around (r, s)-cycle along the σ^1-direction. These two cycles are orthogonal to each other and intersect at least once. Thus, this wrapped supermembrane is expected to give the (p, q)-string [8, 10]. In fact, we shall see below that the (p, q)-string comes out through the double dimensional reduction.

3.2 Matrix regularization and double dimensional reduction

We shall follow the matrix regularization procedure presented in section 2 [22] and then consider the double dimensional reduction [4] with the matrices. One comment is in order: In this paper we do not consider the matrix regularization of the background fields, which play the role of probing the membrane X^M and hence the background fields G_{MN} and A_{MNP} are proportional to the unit matrix in the matrix regularized action. The double dimensional reduction is carried out along the (p, q)-cycle [10], however, we should be careful to really deduce the (p, q)-string. First we should notice the followings. Once we intend to deduce type IIB superstring we shall consider the shrinking volume limit of the 2-torus keeping the ratio of the radii finite,

$$\frac{L_1}{L_2} \equiv g_b : \text{finite.} \quad (L_1, L_2 \to 0) \tag{3.11}$$

and the ratio is the type IIB coupling constant [7]. On the other hand, by using the relations between the 11-dimensional supergravity and 9-dimensional type IIB background fields in eqs. (B.18)–(B.20) [30, 31] we have

$$\sqrt{\frac{G_{88} - \frac{(G_{89})^2}{G_{99}}}{G_{99}}} = e^{-\varphi} = \frac{1}{g_b} = \frac{L_2}{L_1}\,, \tag{3.12}$$

where φ is a background of the type IIB dilaton. Thus eq. (3.12) leads to

$$R_1 = R_2 \equiv R_B\,. \tag{3.13}$$

Then we set (3.13) hereafter in this section.[11]

Next we determine the spacetime directions to align with the worldvolume coordinate, or we fix the gauge. We define X^y and X^z by an SO(2) rotation of the target-space [10]

$$\begin{pmatrix} X^z \\ X^y \end{pmatrix} = O_{(p,q)} \begin{pmatrix} X^9 \\ X^8 \end{pmatrix}, \tag{3.14}$$

[10]We shall see $R_1 = L_1\, e^{-2\phi/3}$ from eq. (B.1) and hence M/IIA-relation, or 11d/IIA-SUGRA-relation, leads to $R_1 = \ell_{11}$ (11-dimensional Planck length).

[11]We do not set eq. (3.13) in section 4.

where

$$O_{(p,q)} = \frac{1}{c_{pq}} \begin{pmatrix} p & q \\ -q & p \end{pmatrix} \equiv \begin{pmatrix} \hat{p} & \hat{q} \\ -\hat{q} & \hat{p} \end{pmatrix} \in \mathrm{SO}(2), \quad c_{pq} \equiv \sqrt{p^2+q^2}\,. \tag{3.15}$$

Then X^z- and X^y-directions are given by

$$X^z = w_1 c_{pq} R_B \sigma^2 + \hat{p}\, Y^1 + \hat{q}\, Y^2 \equiv C_1\, \sigma^2 + Y^z\,, \tag{3.16}$$

$$X^y = \frac{w_2 n_c R_B}{c_{pq}} \sigma^1 - \hat{q}\, Y^1 + \hat{p}\, Y^2 \equiv C_2\, \sigma^1 + Y^y\,, \tag{3.17}$$

and they are aligned with (p,q)- and (r,s)-cycles, respectively. The transverse metric and 3-form are transformed as

$$\tilde{G}_{UV} = G_{MN} \frac{\partial X^M}{\partial X^U} \frac{\partial X^N}{\partial X^V}\,, \quad \tilde{A}_{UVW} = A_{MNP} \frac{\partial X^M}{\partial X^U} \frac{\partial X^N}{\partial X^V} \frac{\partial X^P}{\partial X^W}\,, \tag{3.18}$$

where $U,V,W = 1,2,\cdots,7,y,z$. We shall parametrize $\tilde{G}_{UV}$ as (cf. eq. (B.1))

$$\tilde{G}_{UV} = \begin{pmatrix} \frac{1}{\sqrt{\tilde{G}_{zz}}}\, \tilde{g}_{mn} + \frac{1}{\tilde{G}_{zz}} \tilde{G}_{mz}\tilde{G}_{nz} & \frac{1}{\sqrt{\tilde{G}_{zz}}}\, \tilde{g}_{my} + \frac{1}{\tilde{G}_{zz}} \tilde{G}_{mz}\tilde{G}_{yz} & \tilde{G}_{mz} \\ \frac{1}{\sqrt{\tilde{G}_{zz}}}\, \tilde{g}_{yn} + \frac{1}{\tilde{G}_{zz}} \tilde{G}_{yz}\tilde{G}_{\nu z} & \frac{1}{\sqrt{\tilde{G}_{zz}}}\, \tilde{g}_{yy} + \frac{1}{\tilde{G}_{zz}} \tilde{G}_{yz}\tilde{G}_{yz} & \tilde{G}_{yz} \\ \tilde{G}_{\nu z} & \tilde{G}_{yz} & \tilde{G}_{zz} \end{pmatrix}. \tag{3.19}$$

Here we shall introduce a noncommutativity on spacesheet (2.21)–(2.24) and give a matrix representation as is given in eqs. (2.25)–(2.27), (2.36) and (2.37). The double dimensional reduction on the matrices is carried out by imposing the following conditions on the oscillators (Ξ stands for X^m, Y^y and A) and background fields,

$$Y^z = 0, \quad \partial_{\theta^2}\Xi = 0\,, \tag{3.20}$$

and

$$\partial_{\theta^2}\tilde{G}_{UV} = \partial_{\theta^2}\tilde{A}_{UVW} = 0\,. \tag{3.21}$$

Then the oscillators are reduced to the diagonal matrices

$$\Xi(\theta^1) = \sum_{u_1\in\mathbb{Z}} \sum_{v_1=-M}^{M} \Xi_{(u_1 N+v_1,0)}\, e^{i(u_1 N+v_1)\theta^1/N}\, U^{v_1}\,, \tag{3.22}$$

and the commutators between them automatically vanish. Under the double dimensional reduction, the non-zero components in the action come from the followings,

$$D_\tau X^z = -\frac{C_1}{L}\, \partial_{\theta^1} A\,, \tag{3.23}$$

$$D_\tau X^y = \partial_\tau Y\,, \tag{3.24}$$

$$D_\tau X^m = \partial_\tau X^m\,, \tag{3.25}$$

$$\frac{-i}{2\pi L}[\, X^z, X^y\,] = -\frac{C_1}{L}\, \partial_{\theta^1} Y\,, \tag{3.26}$$

$$\frac{-i}{2\pi L}[\, X^z, X^m\,] = -\frac{C_1}{L}\, \partial_{\theta^1} X^m\,, \tag{3.27}$$

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where $Y \equiv Y^y + (C_2\,\theta^1/N)I_N$. Then the action (3.1) is rewritten by

$$
\begin{aligned}
S = \frac{2\pi TL}{2}\int d\tau\int_0^{2\pi} d\theta^1\ \mathrm{Tr}\Bigg[&\left(\frac{\tilde{g}_{mn}}{\sqrt{\tilde{G}_{zz}}}+\frac{\tilde{G}_{mz}\tilde{G}_{nz}}{\tilde{G}_{zz}}\right)\partial_0 X^m\partial_0 X^n \\
&+2\left(\frac{\tilde{g}_{ym}}{\sqrt{\tilde{G}_{zz}}}+\frac{\tilde{G}_{mz}\tilde{G}_{yz}}{\tilde{G}_{zz}}\right)\partial_0 X^m\partial_0 Y \\
&-\frac{2C_1}{L}\Big(\tilde{G}_{mz}\partial_0 X^m\partial_1 A+\tilde{G}_{yz}\partial_0 Y\partial_1 A\Big)+\tilde{G}_{yy}(\partial_0 Y)^2+\left(\frac{C_1}{L}\right)^2\tilde{G}_{zz}(\partial_1 A)^2 \\
&-\left(\frac{C_1}{L}\right)^2\sqrt{\tilde{G}_{zz}}\Big(\tilde{g}_{yy}(\partial_1 Y)^2+2\tilde{g}_{my}\partial_1 X^m\partial_1 Y+\tilde{g}_{mn}\partial_1 X^m\partial_1 X^n\Big) \\
&+\frac{2C_1}{L}(A_{nmz}\partial_0 X^n\partial_1 X^m+A_{mzy}\epsilon^{ab}\partial_a Y\partial_b X^m)\Bigg], \qquad (3.28)
\end{aligned}
$$

where $a,b=0,1$ and we adopt the notation of $(\partial_0,\partial_1)\equiv(\partial_\tau,\partial_{\theta^1})$ only in section 3. Then, solving the field equation of A and rescaling $\tau\to\tau L/(C_1\sqrt{\tilde{G}_{zz}})$ we obtain the double dimensionally reduced action

$$
\begin{aligned}
S = \frac{2\pi T}{2}\int d\tau\int_0^{2\pi} d\theta^1 C_1\,\mathrm{Tr}\Big[&\eta^{ab}(\tilde{g}_{mn}\partial_a X^m\partial_b X^n+2\tilde{g}_{my}\partial_a X^m\partial_b Y+\tilde{g}_{yy}\partial_a Y\partial_b Y) \\
&+2(\tilde{A}_{nmz}\partial_\tau X^n\partial_1 X^m+\tilde{A}_{mzy}\epsilon^{ab}\partial_a Y\partial_b X^m)\Big]\,. \qquad (3.29)
\end{aligned}
$$

3.3 (p,q)-string from wrapped supermembrane

In this subsection we derive the (p,q)-string action from the reduced supermembrane action in eq. (3.29). The action has an abelian isometry associated with the other compactified Y-direction, we can make a dual transformation as is the case with sigma models. Introducing a variable $\tilde{Y}$, which is seen to be dual to Y, eq. (3.29) can be rewritten in a classically equivalent form

$$
\begin{aligned}
S = \frac{2\pi T}{2}\int d\tau\int_0^{2\pi} d\theta^1\,C_1\,\mathrm{Tr}\Big[&\eta^{ab}(\tilde{g}_{mn}\partial_a X^m\partial_b X^n+2\tilde{g}_{my}\partial_a X^m G_b+\tilde{g}_{yy}G_a G_b) \\
&+2(\tilde{A}_{nmz}\partial_\tau X^n\partial_1 X^m+\tilde{A}_{mzy}\epsilon^{ab}G_a\partial_b X^m)+2\epsilon^{ab}\tilde{Y}\partial_a G_b\Big]\,, \qquad (3.30)
\end{aligned}
$$

since the variation w.r.t. $\tilde{Y}$ leads to $\epsilon^{ab}\partial_a G_b=0$ or $G_a=\partial_a Y$ and hence eq. (3.29) can be reproduced.[12] On the other hand, assuming that all the fields are independent of G_a (or Y), the variation w.r.t. G_a leads to

$$
G_a = \frac{1}{\tilde{g}_{yy}}\Big\{\eta_{ab}\epsilon^{cb}\partial_c\tilde{Y}-\tilde{g}_{my}\partial_a X^m-A_{mzy}(\partial_1 X^m\eta_{0a}-\partial_0 X^m\eta_{a1})\Big\}\,, \qquad (3.31)
$$

[12]We assume that the background fields are independent of $\tilde{Y}$ in eq. (3.30).

and hence we have

$$
\begin{aligned}
S = \frac{2\pi T}{2}\int d\tau \int_0^{2\pi} d\theta^1\, C_1\, \mathrm{Tr}\Bigg[&\left(\tilde{g}_{mn} - \frac{\tilde{g}_{my}\tilde{g}_{ny} - \tilde{A}_{mzy}\tilde{A}_{nzy}}{\tilde{g}_{yy}}\right)\eta^{ab}\partial_a X^m \partial_b X^n \\
&+ 2\,\frac{\tilde{A}_{mzy}}{\tilde{g}_{yy}}\,\eta^{ab}\partial_a X^m \partial_b \tilde{Y} + \frac{1}{\tilde{g}_{yy}}\,\eta^{ab}\partial_a \tilde{Y}\partial_b \tilde{Y} \\
&+ \left(\tilde{A}_{mnz} + 2\,\frac{\tilde{A}_{mzy}\tilde{g}_{ny}}{\tilde{g}_{yy}}\right)\epsilon^{ab}\partial_a X^m \partial_b X^n + 2\,\frac{\tilde{g}_{my}}{\tilde{g}_{yy}}\epsilon^{ab}\partial_a \tilde{Y}\partial_b X^m\Bigg]\,. \qquad (3.32)
\end{aligned}
$$

Now that we consider T-dual for the background fields in eq. (3.29) (or eq. (3.32)). Since we regard X^9 (*not* X^z) as the 11th direction, we should take T-dual along the X^8-direction to transform type IIA superstring theory to type IIB superstring theory. Then we can rewrite the background fields in terms of those of the type IIB supergravity as follows [30, 31](cf. appendix B),

$$
\tilde{g}_{my} = \frac{B^{(pq)}_{8m}}{\jmath_{88}}\,, \qquad (3.33)
$$

$$
\tilde{g}_{yy} = \frac{1}{\jmath_{88}\sqrt{(\hat{p}+\hat{q}l)^2 + e^{-2\varphi}\hat{q}^2}}\,, \qquad (3.34)
$$

$$
\tilde{g}_{mn} = \sqrt{(\hat{p}+\hat{q}l)^2 + e^{-2\varphi}\hat{q}^2}\left(\jmath_{mn} - \frac{\jmath_{8m}\jmath_{8n}}{\jmath_{88}} + \frac{B^{(pq)}_{8m}B^{(pq)}_{8n}}{\jmath_{88}}\right), \qquad (3.35)
$$

$$
\tilde{A}_{mnz} = \sqrt{(\hat{p}+\hat{q}l)^2 + e^{-2\varphi}\hat{q}^2}\left(B^{(pq)}_{mn} + \frac{2}{\jmath_{88}}B^{(pq)}_{8[m}\jmath_{n]8}\right), \qquad (3.36)
$$

$$
\tilde{A}_{myz} = -\frac{\jmath_{8m}}{\jmath_{88}} = \sqrt{(\hat{p}+\hat{q}l)^2 + e^{-2\varphi}\hat{q}^2}\;\tilde{g}_{yy}\jmath_{8m}\,, \qquad (3.37)
$$

where $B^{(1)}_{IJ}$ and $B^{(2)}_{IJ}$ are the NSNS and RR second-rank antisymmetric tensors, respectively, $\jmath_{IJ}$ are the metric in type IIB supergravity, $l = G_{89}/G_{99} = A_8$ and

$$
B^{(pq)}_{IJ} = \frac{\hat{p}\,B^{(1)}_{IJ} + \hat{q}\,B^{(2)}_{IJ}}{\sqrt{(\hat{p}+\hat{q}l)^2 + e^{-2\varphi}\hat{q}^2}}\,, \qquad (3.38)
$$

where $I, J = 1, 2, \cdots, 8$. Then, plugging these equations into eq. (3.32) we have

$$
\begin{aligned}
S = \frac{2\pi T}{2}\int d\tau \int_0^{2\pi} &d\theta^1\, C_1 \sqrt{(\hat{p}+\hat{q}l)^2 + e^{-2\varphi}\hat{q}^2} \\
&\times \mathrm{Tr}\Big[\eta^{ab}\partial_a \tilde{X}^I \partial_b \tilde{X}^J \jmath_{IJ} + \epsilon^{ab}\partial_a \tilde{X}^I \partial_b \tilde{X}^J B^{(pq)}_{IJ}\Big]\,, \qquad (3.39)
\end{aligned}
$$

where we have defined $\tilde{X}^I \equiv (X^m, \tilde{Y})$. Once we regard X^9 as the 11th direction, the type IIA string tension T_s is given by $2\pi L_1 T/\sqrt{G_{99}}$ [8] since the 11-dimensional metric G_{MN} is converted to the type IIA metric g_{IJ} by the relation $G_{IJ} = g_{IJ}/\sqrt{G_{99}}$. Also, if we assume that l and φ are constant and hence $e^{\varphi} = g_b$, we have

$$
2\pi T C_1 \sqrt{(\hat{p}+\hat{q}l)^2 + e^{-2\varphi}\hat{q}^2} = w_1 T_s \sqrt{(p+ql)^2 + e^{-2\varphi}q^2} \equiv w_1 T_{pq} \qquad (3.40)
$$

where T_{pq} is the tension of a (p,q)-string in type IIB superstring theory [8]. In particular, we see that both of the NSNS and RR antisymmetric tensors couple to $\tilde{X}^I$ in eq. (3.39), which implies that the reduced action (3.39) is, in fact, that of the (p, q)-strings. Note that w_1 is just the number of copies of the resulting (p, q)-string. If we set q to be zero and hence take $(p, q, r, s) = (1, 0, 0, 1)$, we have the fundamental strings in type IIB superstring theory. On the other hand, $(p, q, r, s) = (0, 1, 1, 0)$ leads to the strings which couple minimally with the RR B-field, i.e., the D-strings.

4. Matrix-regularized action in curved background

In this section we perform the matrix regularization on the wrapped supermembrane in curved background by adopting a suitable choice of matrix representation. As we mentioned before, the background fields are to be proportional to the unit matrix in the matrix regularization here. Then we derive the standard form of (2+1)-dimensional super Yang-Mills action in the curved background.

4.1 Standard form of super Yang-Mills action

Let us start with the wrapped supermembrane (3.1) where X^9, X^8, X^m and A are given by eqs. (3.7), (3.8), (2.6) and (2.7), respectively. In this section we assume that the background metric is block-diagonal

$$G_{MN} = \begin{pmatrix} G_{mn} & 0 \\ 0 & G_{uv} \end{pmatrix} . \tag{4.1}$$

We shall perform the matrix regularization. The procedure is the same as before (cf. subsection 2.1), however, we should be careful in the choice of matrix representation of the algebra. In this case by using the general matrix representation in eqs. (2.72)–(2.74), we shall search for the set of parameters which makes the matrix representations of X^9 and X^8 similar to eqs. (2.30) and (2.31), respectively. In fact, we find that the choice of the parameters

$$\begin{aligned} c^1 &= -\frac{2\pi i s}{w_1 n_c}, & c^2 &= \frac{2\pi i r}{w_1 n_c}, & e^1 &= \frac{2\pi i q}{w_2 n_c}, & e^2 &= -\frac{2\pi i p}{w_2 n_c}, \\ d_1 &= -\frac{w_2 r}{N}, & d_2 &= 0, & f_1 &= \frac{w_1 p}{N}, & f_2 &= 0, \end{aligned} \tag{4.2}$$

leads to the matrix representation

$$X^9(\sigma^1, \sigma^2) \to -2\pi i R_1 \partial_{\theta^1} I_N + Y^1(\theta^1, \theta^2), \tag{4.3}$$

$$X^8(\sigma^1, \sigma^2) \to -2\pi i R_2 \partial_{\theta^2} I_N + \frac{w_1 w_2 n_c R_2}{N} \theta^1 I_N + Y^2(\theta^1, \theta^2), \tag{4.4}$$

$$X^m(\sigma^1, \sigma^2) \to X^m(\theta^1, \theta^2), \tag{4.5}$$

$$A(\sigma^1, \sigma^2) \to A(\theta^1, \theta^2), \tag{4.6}$$

and the oscillation modes of the $N \times N$ matrices are give by ($\Xi\,(\theta^1, \theta^2)$ stands for $Y^1\,(\theta^1, \theta^2)$, $Y^2\,(\theta^1, \theta^2)$, $X^m\,(\theta^1, \theta^2)$ and $A\,(\theta^1, \theta^2)$)

$$\Xi(\theta^1, \theta^2) = \sum_{u_1, u_2 \in \mathbb{Z}} \sum_{v_1, v_2 = -M}^{M} \Xi_{(u_1 N + v_1, u_2 N + v_2)}\, e^{i K_1 \theta^1 / N} e^{-i K_2 \theta^2 / N}\, \lambda^{-v_1 v_2 / 2}\, V^{v_2} U^{v_1}, \tag{4.7}$$

where

$$K_1 = w_1 p(u_1 N + v_1) - w_2 r(u_2 N + v_2)\,, \quad K_2 = -w_1 q(u_1 N + v_1) + w_2 s(u_2 N + v_2)\,. \quad (4.8)$$

Then, Ξ satisfies the boundary conditions

$$\Xi\left(\theta^1 + \frac{2\pi s}{w_1 n_c},\, \theta^2 - \frac{2\pi r}{w_1 n_c}\right) = V\,\Xi(\theta^1, \theta^2)\, V^\dagger\,, \quad (4.9)$$

$$\Xi\left(\theta^1 - \frac{2\pi q}{w_2 n_c},\, \theta^2 + \frac{2\pi p}{w_2 n_c}\right) = U\,\Xi(\theta^1, \theta^2)\, U^\dagger\,, \quad (4.10)$$

or

$$\Xi(\theta^1 + 2\pi, \theta^2) = U^{w_2 r} V^{w_1 p}\, \Xi(\theta^1, \theta^2)\, (U^{w_2 r} V^{w_1 p})^\dagger\,, \quad (4.11)$$

$$\Xi(\theta^1, \theta^2 + 2\pi) = U^{w_2 s} V^{w_1 q}\, \Xi(\theta^1, \theta^2)\, (U^{w_2 s} V^{w_1 q})^\dagger\,, \quad (4.12)$$

which express that the supermembrane wraps around (p, q)- and (r, s)-cycles. Furthermore, since we have

$$[\,\sigma^1, \sigma^2\,]_* \to [\,c^i \partial_{\theta^i} + d_i \theta^i, e^i \partial_{\theta^i} + f_i \theta^i\,] = \frac{2\pi i}{N}\,, \quad (4.13)$$

$$\frac{1}{N} \int_F d\theta^1 d\theta^2\, \mathrm{Tr}\, I_N = \frac{(2\pi)^2}{w_1 |w_2| n_c}\,, \quad (4.14)$$

where F is a parallelogram generated by the two vectors, $(2\pi s/(w_1 n_c), -2\pi r/(w_1 n_c))$ and $(-2\pi q/(w_2 n_c), 2\pi p/(w_2 n_c))$, the Poisson bracket and the double integral are represented as

$$\{\,\cdot\,,\,\cdot\,\} \to -i\,\frac{N}{2\pi}\,[\,\cdot\,,\,\cdot\,]\,, \quad (4.15)$$

$$\int_0^{2\pi} d\sigma^1 d\sigma^2 * \to \frac{w_1 |w_2| n_c}{N} \int_F d\theta^1 d\theta^2\ \mathrm{Tr}[\,*\,]\ \left(= \frac{1}{N} \int_0^{2\pi} d\theta^1 d\theta^2\ \mathrm{Tr}[\,*\,]\right)\,. \quad (4.16)$$

The first two terms of the action (3.1) are given by, with rescaling $\tau \to \tau/N$,

$$\begin{aligned} S_{2+1} = \frac{w_1 |w_2| n_c L T}{2} \int d\tau \int_F d\theta^1 d\theta^2\ \mathrm{Tr}\Big[& G_{99}(F_{\tau\theta^1})^2 + 2G_{89} F_{\tau\theta^1} F_{\tau\theta^2} \\ & + G_{88}(F_{\tau\theta^2})^2 - V_{T^2}(F_{\theta^1\theta^2})^2 + (D_\tau X^m)^2 - G_{99}(D_{\theta^1} X^m)^2 \\ & - 2G_{89} G_{mn} D_{\theta^1} X^m D_{\theta^2} X^n - G_{88}(D_{\theta^2} X^m)^2 + \frac{1}{2(2\pi L)^2}\,[\,X^m, X^n\,]^2\Big], \end{aligned} \quad (4.17)$$

where

$$F_{\tau\theta^1} = \partial_\tau Y^1 - \frac{R_1}{L}\, \partial_{\theta^1} A + \frac{i}{2\pi L}\,[\,A, Y^1\,]\,, \quad (4.18)$$

$$F_{\tau\theta^2} = \partial_\tau Y^2 - \frac{R_2}{L}\, \partial_{\theta^2} A + \frac{i}{2\pi L}\,[\,A, Y^2\,]\,, \quad (4.19)$$

$$F_{\theta^1\theta^2} = \frac{w_1 w_2 n_c R_1 R_2}{NL} I_N + \frac{R_1}{L}\, \partial_{\theta^1} Y^2 - \frac{R_2}{L}\, \partial_{\theta^2} Y^1 + \frac{i}{2\pi L}\,[\,Y^1, Y^2\,]\,, \quad (4.20)$$

$$D_\tau X^m = \partial_\tau X^m + \frac{i}{2\pi L}\,[\,A, X^m\,]\,, \quad (4.21)$$

$$D_{\theta^1} X^m = \frac{R_1}{L}\, \partial_{\theta_1} X^m + \frac{i}{2\pi L}\,[\,Y^1, X^m\,]\,, \quad (4.22)$$

$$D_{\theta^2} X^m = \frac{R_2}{L}\, \partial_{\theta_2} X^m + \frac{i}{2\pi L}\,[\,Y^2, X^m\,]\,. \quad (4.23)$$

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As before, we rewrite the matrix regularized action by introducing some dimensionful constants to adjust the mass dimensions of the fields and the parameters,

$$Y^1(\theta^1,\theta^2) \to \hat{\alpha} A_1(x^1,x^2)\,, \tag{4.24}$$
$$Y^2(\theta^1,\theta^2) \to \hat{\alpha} A_2(x^1,x^2)\,, \tag{4.25}$$
$$X^m(\theta^1,\theta^2) \to \hat{\alpha} \phi^m(x^1,x^2)\,, \tag{4.26}$$
$$A(\theta^1,\theta^2) \to \hat{\alpha} A_0(x^1,x^2)\,, \tag{4.27}$$
$$\theta^1 \to x^1/\hat{\Sigma}_1\,, \tag{4.28}$$
$$\theta^2 \to x^2/\hat{\Sigma}_2\,, \tag{4.29}$$
$$\tau \to x^0/\hat{\Sigma}\,. \tag{4.30}$$

where $\hat{\alpha}$ has mass dimension -2 and $\hat{\Sigma}_1$, $\hat{\Sigma}_2$ and $\hat{\Sigma}$ have mass dimension -1 as in eqs. (2.45)–(2.51). Then, in the same way as in subsection 2.1, in order to have the standard form of the super Yang-Mills action, we should set

$$\hat{\Sigma} = \frac{\hat{\alpha}}{2\pi L}\,, \tag{4.31}$$
$$\hat{\Sigma}_1 = \frac{\hat{\alpha}}{2\pi R_1}\,, \tag{4.32}$$
$$\hat{\Sigma}_2 = \frac{\hat{\alpha}}{2\pi R_2}\,. \tag{4.33}$$

Thus eqs. (4.17)–(4.23) are reduced to (we put $\hat{\alpha} = \alpha$)

$$S_{2+1} = \frac{w_1|w_2|n_c}{g_{YM}^2} \int dx^0 \int_{\mathcal{F}} dx^1 dx^2 \sqrt{-\det \mathcal{G}_{\alpha\beta}}\ \mathrm{Tr}\Big[-\frac{1}{4}\,\mathcal{G}^{\alpha\beta}\mathcal{G}^{\gamma\delta} F_{\alpha\gamma}F_{\beta\delta} - \frac{1}{2}\,\mathcal{G}^{\alpha\beta} D_\alpha\phi^m D_\beta\phi^n G_{mn} + \frac{1}{4}\, G_{mp}G_{nq}[\,\phi^m,\phi^n\,][\,\phi^p,\phi^q\,]\Big], \tag{4.34}$$
$$F_{\alpha\beta} = f_{\alpha\beta} + \partial_\alpha A_\beta - \partial_\beta A_\alpha + i[\,A_\alpha, A_\beta\,]\,, \tag{4.35}$$
$$D_\alpha\phi^m = \partial_\alpha\phi^m + i[\,A_\alpha,\phi^m\,]\,, \tag{4.36}$$

where $\alpha,\beta,\gamma,\delta = 0,1,2$, the worldvolume metric $\mathcal{G}_{\alpha\beta}$ is

$$\mathcal{G}_{\alpha\beta} = \left(\mathcal{G}^{\alpha\beta}\right)^{-1}, \quad \mathcal{G}^{\alpha\beta} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & G_{99} & G_{89} \\ 0 & G_{89} & G_{88} \end{pmatrix}, \tag{4.37}$$

the constant magnetic flux $f_{\alpha\beta}$ is

$$f_{01} = f_{02} = 0, \quad f_{12} = \frac{w_1 w_2 n_c}{2\pi N \hat{\Sigma}_1\hat{\Sigma}_2}\, I_N\,, \tag{4.38}$$

and the region $\mathcal{F}$ in the spacesheet (x^1,x^2) is a parallelogram spanned by the two vectors $(2\pi\hat{\Sigma}_1 s/(w_1 n_c), -2\pi\hat{\Sigma}_2 r/(w_1 n_c))$ and $(-2\pi\hat{\Sigma}_1 q/(w_2 n_c), 2\pi\hat{\Sigma}_2 p/(w_2 n_c))$. The matrices satisfy the boundary conditions (Ξ stands for A_α and ϕ^m)

$$\Xi\left(x^1 + \frac{2\pi\hat{\Sigma}_1 s}{w_1 n_c},\, x^2 - \frac{2\pi\hat{\Sigma}_2 r}{w_1 n_c}\right) = V\,\Xi(x^1,\,x^2)\,V^\dagger\,, \tag{4.39}$$
$$\Xi\left(x^1 - \frac{2\pi\hat{\Sigma}_1 q}{w_2 n_c},\, x^2 + \frac{2\pi\hat{\Sigma}_2 p}{w_2 n_c}\right) = U\,\Xi(x^1,\,x^2)\,U^\dagger\,, \tag{4.40}$$

or

$$\Xi(x^1 + 2\pi\hat{\Sigma}_1,\, x^2) = U^{w_2 r} V^{w_1 p}\, \Xi(x^1,\, x^2)\, (U^{w_2 r} V^{w_1 p})^\dagger\,, \tag{4.41}$$

$$\Xi(x^1,\, x^2 + 2\pi\hat{\Sigma}_2) = U^{w_2 s} V^{w_1 q}\, \Xi(x^1,\, x^2)\, (U^{w_2 s} V^{w_1 q})^\dagger\,. \tag{4.42}$$

Note that the gauge coupling constant g_{YM} is the same as in the flat background case in eq. (2.70). The third term of the action (3.1) is reduced to

$$\begin{aligned} S_f = \frac{w_1|w_2|n_c}{2g_{YM}^2} \int dx^0 \int_{\mathcal{F}} dx^1 dx^2 \sqrt{-\det \mathcal{G}_{\alpha\beta}}\ \mathrm{Tr}\Big[& -iD_0\phi^l \hat{A}_{lmn}[\,\phi^m, \phi^n\,] \\ & + 2D_0\phi^m (D_1\phi^n \hat{A}_{mn9} + D_2\phi^n \hat{A}_{mn8}) - i(F_{01}\hat{A}_{mn9} + F_{02}\hat{A}_{mn8})[\,\phi^m, \phi^n\,] \\ & + \epsilon^{\alpha\beta\gamma} F_{\alpha\beta} D_\gamma \phi^m\, \hat{A}_{m89} \Big]\,. \end{aligned} \tag{4.43}$$

One comment is in order: In this subsection we have derived the matrix regularized action by using the general matrix representation in eqs. (2.72)–(2.74). Of course, the same matrix regularized action can be obtained when we first transform X^9 and X^8 in eqs. (3.7) and (3.8) by a $GL(2, \mathbb{R})$ matrix as

$$\begin{pmatrix} X^9 \\ X^8 \end{pmatrix} \to \begin{pmatrix} \tilde{X}^9 \\ \tilde{X}^8 \end{pmatrix} = \frac{1}{n_c} \begin{pmatrix} s & -rR_1/R_2 \\ -qR_2/R_1 & p \end{pmatrix} \begin{pmatrix} X^9 \\ X^8 \end{pmatrix} \tag{4.44}$$

and then perform the regularization as in subsection 2.1.

4.2 Duality

In this subsection we examine the symmetry of the matrix-regularized action, which is the sum of S_{2+1} (4.34) and S_f (4.43),

$$S_{MR} = S_{2+1} + S_f\,. \tag{4.45}$$

If we regard x^i as the local coordinate of a general two-dimensional manifold assuming $\mathcal{F} \to \mathbb{R}^2$ or $R_1, R_2 \to 0$, S_{MR} is formally invariant under the following two-dimensional general coordinate transformation $GC(2, \mathbb{R})$,

$$x^i \to \tilde{x}^i = f^i(x)\,, \tag{4.46}$$

$$\mathcal{G}^{ij}(x) \to \tilde{\mathcal{G}}^{ij}(\tilde{x}) = M^i{}_k M^j{}_l\, \mathcal{G}^{kl}(x)\,, \tag{4.47}$$

$$A_i(x) \to \tilde{A}_i(\tilde{x}) = A_j(x)(M^{-1})^j{}_i\,, \tag{4.48}$$

$$A_0(x) \to \tilde{A}_0(\tilde{x}) = A_0(x)\,, \tag{4.49}$$

$$\begin{pmatrix} A_{mn9} \\ A_{mn8} \end{pmatrix}(x) \to \begin{pmatrix} \tilde{A}_{mn9} \\ \tilde{A}_{mn8} \end{pmatrix}(\tilde{x}) = M \begin{pmatrix} A_{mn9} \\ A_{mn8} \end{pmatrix}(x), \tag{4.50}$$

$$A_{m89}(x) \to \tilde{A}_{m89}(\tilde{x}) = (\det M)\, A_{m89}(x)\,, \tag{4.51}$$

$$A_{mnp}(x) \to \tilde{A}_{mnp}(\tilde{x}) = A_{mnp}(x)\,, \tag{4.52}$$

$$\phi^m(x) \to \tilde{\phi}^m(\tilde{x}) = \phi^m(x)\,, \quad (i, j, k, l = 1, 2) \tag{4.53}$$

where

$$M^i{}_j \ \left(= \frac{\partial \tilde{x}^i}{\partial x^j}\right) \in GL(2,\mathbb{R})\,. \tag{4.54}$$

Note that eq. (4.48) corresponds to $GL(2,\mathbb{R})$-transformation on the (target-space) 8-9 plane in membrane theory.

Let us consider SL$(2,\mathbb{R})$ transformation, which is a subgroup of the $GC(2,\mathbb{R})$,

$$\tilde{\mathcal{G}}^{ij}(x) = \Lambda^i{}_k\,\Lambda^j{}_l\,\mathcal{G}^{kl}(\Lambda^{-1}x)\,, \quad \tilde{A}_i(x) = (\Lambda^{-1})^j{}_i\,A_j(\Lambda^{-1}x)\,, \quad \tilde{\phi}^m(x) = \phi^m(\Lambda^{-1}x)\,, \quad \text{etc.}\,, \tag{4.55}$$

where Λ is a constant matrix of SL$(2,\mathbb{R})$ parametrized by

$$\Lambda = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathrm{SL}(2,\mathbb{R})\,. \tag{4.56}$$

It was shown that the type IIB superstring SL$(2,\mathbb{R})$ duality (at the classical level) can be realized as the SL$(2,\mathbb{R})$ target-space rotation of 11-dimensional theory in the effective action [30]. In particular, we can easily check that the SL$(2,\mathbb{R})$ transformation can be rewritten as (cf. appendix B)

$$\tilde{\tau} = \frac{c+d\tau}{a+b\tau}\,, \quad \tilde{\jmath}_{IJ} = |a+b\tau|\,\jmath_{IJ}\,, \quad \begin{pmatrix} \tilde{B}^{(1)}_{IJ} \\ \tilde{B}^{(2)}_{IJ} \end{pmatrix} = \Lambda \begin{pmatrix} B^{(1)}_{IJ} \\ B^{(2)}_{IJ} \end{pmatrix}\,, \quad \tilde{D}_{mnp8} = D_{mnp8}\,, \quad \text{etc.}, \tag{4.57}$$

where $\tau \equiv l + ie^{-\varphi}$ is the moduli fields of a 2-torus and $I, J = 1, \cdots, 8$.[13] Notice that this transformation is, in fact, corresponds to the type IIB superstring SL$(2,\mathbb{R})$ duality. For example, we can see that when $a = d = 0$, $b = -c = -1$ and $l = 0$, the SL$(2,\mathbb{R})$ transformation is reduced to the strong-weak duality $\tilde{\varphi} = -\varphi$, or $e^{\varphi} \to 1/e^{\varphi}$ in the type IIB superstring theory, which will be seen in the followings.

Next, we shall examine the type IIB string duality. Let us consider two 2-tori of (L_1, L_2) and $(\tilde{L}_1, \tilde{L}_2)$ whose metrics are G_{uv} and $\tilde{G}_{uv}$, respectively (see (4.1)). Then, following the procedure in the previous subsection, we shall obtain the matrix regularized action of the standard form of the super Yang-Mills action (4.34) in each case. We regard that those two are to be related by a SL$(2,\mathbb{R})$ transformation (4.55)–(4.56). Then, once we consider the reduction to type IIB superstring with each 2-torus, we shall put $R_1 = R_2 \equiv R_B$ and $\tilde{R}_1 = \tilde{R}_2 \equiv \tilde{R}_B$ as in eq. (3.13) and hence we find that the string couplings are related by

$$\tilde{g}_b = \frac{\tilde{L}_1}{\tilde{L}_2} = \frac{|\tilde{G}_{99}|}{\sqrt{\det \tilde{G}_{uv}}} = |a+b\tau|^2 \frac{|G_{99}|}{\sqrt{\det G_{uv}}} = |a+b\tau|^2\,\frac{L_1}{L_2} = |a+b\tau|^2\,g_b\,. \tag{4.58}$$

Furthermore, since both R_1 and $\tilde{R}_1$ correspond to ℓ_{11} (cf. footnote 10), the oscillation parts of the matrices (A_α and ϕ^m) in the action are related by the SL$(2,\mathbb{R})$ transformation as

[13] In this section, we put the spacetime metric in a block diagonal form eq. (4.1) to obtain the standard form of Yang-Mills action eqs. (4.34) and (4.43) through the matrix regularization of wrapped membrane. Then, eq. (4.1) leads to $B^{(1)}_{m8} = B^{(2)}_{m8} = 0$ in type IIB superstring variables (cf. appendix B). However, if we do not stick to the standard form of Yang-Mills action and carry out the matrix regularization by keeping the off-diagonal block non-zero, we shall see that two-dimensional general coordinate transformation $GC(2,\mathbb{R})$ leads to the third equation of (4.57) with non-zero $B^{(1)}_{m8}$ and $B^{(2)}_{m8}$, in general.

(cf. (4.7), (4.8), (4.28) and (4.29))

$$\exp\left[\frac{i}{N}\frac{K_1x^1-K_2x^2}{\hat{\Sigma}_R}\right]\leftrightarrow\exp\left[\frac{i}{N}\frac{K_1y^1-K_2y^2}{\tilde{\hat{\Sigma}}_R}\right]\quad(y^i=(\Lambda^{-1})^i{}_j\,x^j)$$
$$=\exp\left[\frac{i}{N}\frac{(dK_1+cK_2)x^1-(bK_1+aK_2)x^2}{\hat{\Sigma}_R}\right],\tag{4.59}$$

where $\hat{\Sigma}_R=\alpha/(2\pi\tilde{R}_R)$ and $\tilde{\hat{\Sigma}}_R=\alpha/(2\pi\tilde{R}_R)$. In eq. (4.59) $R_B=\tilde{R}_B$ has been used and hence $\hat{\Sigma}_1=\hat{\Sigma}_2\equiv\hat{\Sigma}_R=\tilde{\hat{\Sigma}}_1=\tilde{\hat{\Sigma}}_2$. We may rewrite (4.59) as the transformation of K_i's,

$$K_1\to\tilde{K}_1=dK_1+cK_2\,,\tag{4.60}$$
$$K_2\to\tilde{K}_2=bK_1+aK_2\,.\tag{4.61}$$

Since we expect $\tilde{K}_i\in\mathbb{Z}$, we shall put restrictions on the parameters,

$$a,b,c,d\in\mathbb{Z}\leftrightarrow\Lambda\in\mathrm{SL}(2,\mathbb{Z})\tag{4.62}$$

Eqs. (4.60)–(4.61) can be rewritten as the transformation of (p,q,r,s)

$$(\,p\;\;q\,)\to(\,\tilde{p}\;\;\tilde{q}\,)=(\,p\;\;q\,)\begin{pmatrix}d&-b\\-c&a\end{pmatrix}=(\,p\;\;q\,)\;\Lambda^{-1}\,,\tag{4.63}$$
$$(\,r\;\;s\,)\to(\,\tilde{r}\;\;\tilde{s}\,)=(\,r\;\;s\,)\;\Lambda^{-1}\,.\tag{4.64}$$

This means that the matrices $\Xi_{(\boldsymbol{p})}(\Lambda^{-1}x)$ can be written by $\Xi_{(\tilde{\boldsymbol{p}})}(x)$, where we have added the suffices in order to distinguish the parameters (p,q,r,s) in the matrices (cf. (4.7)–(4.8)). Here, we should make a comment on the double dimensional reduction. From eqs. (3.14)–(3.15), (4.24)–(4.25) and (4.7)–(4.8) we shall see that the conditions of the double dimensional reduction which corresponds to eqs. (3.20)–(3.21) are given by (putting $R_1=R_2$)

$$pA_1(x)+qA_2(x)=0\,,\quad(q\partial_1-p\partial_2)\Phi(x)=0\,,\tag{4.65}$$

where Φ stands for all the matrices and background fields. Similarly, in the $\mathrm{SL}(2,\mathbb{R})$-transformed frame the double dimensional reduction should be done by (with $\tilde{R}_1=\tilde{R}_2$)

$$\tilde{p}\tilde{A}_1(x)+\tilde{q}\tilde{A}_2(x)=0\,,\quad(\tilde{q}\partial_1-\tilde{p}\partial_2)\tilde{\Phi}(x)=0\,.\tag{4.66}$$

As was mentioned before, if we choose a $\mathrm{SL}(2,\mathbb{R})$-matrix

$$\Lambda=\begin{pmatrix}0&-1\\1&0\end{pmatrix}\in\mathrm{SL}(2,\mathbb{R})\,,\tag{4.67}$$

eq. (4.58) becomes

$$g_b\to\tilde{g}_b=|\tau|^2\,g_b=l^2g_b+g_b^{-1}\,,\tag{4.68}$$

and eq. (4.63) leads to

$$(\,p\;\;q\,)\to(\,\tilde{p}\;\;\tilde{q}\,)=(\,-q\;\;p\,)\,.\tag{4.69}$$

This indicates that in type IIB superstring the system of a (p,q)-string with the string coupling g_b is dual to that of a $(-q,p)$-string with $g_b^{-1}+l^2 g_b$. This can be seen through the terms $L_B(x) \equiv 2D_0\phi^m(D_1\phi^n A_{mn9}+D_2\phi^n A_{mn8})$ in S_f (4.43). In the $\mathrm{SL}(2,\mathbb{R})$-transformed frame, L_B is given by

$$\begin{aligned}
\tilde{L}_B(x) &= 2\tilde{D}_0\tilde{\phi}^m(x)(\tilde{D}_1\tilde{\phi}^n(x)\tilde{A}_{mn9}(x)+\tilde{D}_2\tilde{\phi}^n(x)\tilde{A}_{mn8}(x)) \\
&= 2\tilde{D}_0\tilde{\phi}^m\left\{(\,\tilde{D}_1\tilde{\phi}^n\;\;\tilde{D}_2\tilde{\phi}^n\,)\Lambda_{\tilde{p}\tilde{q}}^{-1}\,\Lambda_{\tilde{p}\tilde{q}}\begin{pmatrix}\tilde{A}_{mn9}(x)\\ \tilde{A}_{mn8}(x)\end{pmatrix}\right\} \\
&\to \frac{1}{n_c}2\tilde{D}_0\tilde{\phi}^m\left\{(\,(\tilde{s}\partial_1-\tilde{r}\partial_2)\tilde{\phi}^n\;\;0\,)\begin{pmatrix}\tilde{p}\tilde{B}^{(1)}_{mn}+\tilde{q}\tilde{B}^{(2)}_{mn}\\ \tilde{r}\tilde{B}^{(1)}_{mn}+\tilde{s}\tilde{B}^{(2)}_{mn}\end{pmatrix}\right\} \\
&= \frac{2}{n_c}\,\partial_0\tilde{\phi}^m(\tilde{s}\partial_1-\tilde{r}\partial_2)\tilde{\phi}^n\,(\tilde{p}\tilde{B}^{(1)}_{mn}+\tilde{q}\tilde{B}^{(2)}_{mn})\,.
\end{aligned} \tag{4.70}$$

One comment is in order: For a constant $GL(2,\mathbb{R})$ matrix $M=(M^i{}_j)$ in eq. (4.54), the maps corresponding to eqs. (4.58) and (4.63) are given by

$$g_b \to \tilde{g}_b = \frac{|M^1{}_1+M^1{}_2\,\tau|^2}{|\det M|}\,g_b\,, \tag{4.71}$$

$$(\,p\;\;q\,)\to(\,\tilde{p}\;\;\tilde{q}\,)=(\,p\;\;q\,)\;M^{-1}\,. \tag{4.72}$$

Finally in this subsection, we refer to two specific transformations, which are not the elements in the $\mathrm{SL}(2,\mathbb{R})$ subgroup. First we examine the X^8-reflection

$$M=\begin{pmatrix}1&0\\0&-1\end{pmatrix}\in Z_2\subset O(2)\,. \tag{4.73}$$

Then the corresponding type IIB superstring duality is given by

$$\begin{aligned}
&\tilde{\tau}=-\bar{\tau}\,, \qquad\qquad \tilde{\jmath}_{mn}=\jmath_{mn}\,,\quad \tilde{\jmath}_{8m}=-\jmath_{8m}\,,\quad \tilde{\jmath}_{88}=\jmath_{88}\,,\\
&\begin{pmatrix}\tilde{B}^{(1)}_{IJ}\\ \tilde{B}^{(2)}_{IJ}\end{pmatrix}=\begin{pmatrix}B^{(1)}_{IJ}\\ -B^{(2)}_{IJ}\end{pmatrix}\,,\quad \tilde{D}_{mnp8}=D_{mnp8}\,.
\end{aligned} \tag{4.74}$$

In this case the type IIB string coupling is invariant and (p,q)-string is mapped to $(p,-q)$-string under the duality. Note that D_{8mnp} is invariant under the reflection of X^8 since we have respected the symmetry of the membrane theory. (See the discussion in ref. [31].)

Similarly, we consider the 8-9 flip

$$M=\begin{pmatrix}0&1\\1&0\end{pmatrix}\in Z_2\subset O(2)\,. \tag{4.75}$$

Then the type IIB superstring duality is given by

$$\begin{aligned}
&\tilde{\tau}=\frac{1}{\bar{\tau}}\,, \qquad\qquad \tilde{\jmath}_{mn}=|\tau|\,\jmath_{mn}\,,\quad \tilde{\jmath}_{8m}=-|\tau|\,\jmath_{8m}\,,\quad \tilde{\jmath}_{88}=|\tau|\,\jmath_{88}\,,\\
&\begin{pmatrix}\tilde{B}^{(1)}_{IJ}\\ \tilde{B}^{(2)}_{IJ}\end{pmatrix}=\begin{pmatrix}B^{(2)}_{IJ}\\ B^{(1)}_{IJ}\end{pmatrix}\,,\quad \tilde{D}_{mnp8}=D_{mnp8}\,.
\end{aligned} \tag{4.76}$$

This implies the (p,q)-string $\leftrightarrow$ (q,p)-string and (in $l=0$ case) the strong-weak $g_b\leftrightarrow g_b^{-1}$ duality.

5. Summary and discussion

In this paper we have studied matrix regularization of the wrapped supermembrane compactified on a 2-torus. We have adopted the lightcone wrapped supermembrane compactified on T^2 in the curved background and the wrapping is characterized by two mutually prime integers (p, q). We have followed the matrix regularization procedure [22] and also applied the double dimensional reduction technique [4] properly to the matrix-regularized action as was done in the continuous case [10]. We have succeeded in deducing explicitly the bosonic sector of the matrix regularized (p, q)-string action in eq. (3.39) directly from the wrapped membrane. A BPS saturated classical solution of the (p, q)-string action (3.39) is valid irrespective of the value of the string coupling g_b, however the valid region to treat the (p, q)-string perturbatively is still obscure and is deserved to be investigated.[14] We have also deduced the (2+1)-dimensional super Yang-Mills theory in a curved background and then we have seen that it really has the symmetries which are related to string duality [32, 1].

In this paper we have considered only classically the limit of vanishing volume of the 2-torus with the wrapped supermembrane and it is, of course, important to investigate it quantum mechanically. In fact, quantum mechanical justification of the double dimensional reduction was studied in refs. [19, 33]. In those references, the Kaluza-Klein modes associated with the ρ-coordinate were not removed classically, but they were integrated in the path integral formulation of the wrapped supermembrane theory. However, it is still in the beginnings of the quantum mechanical study, and it deserves to be investigated further with the results in this paper.

A. Notation

The target-space indices;

$$M, N, P, Q = 1, 2, \cdots, 7, 8, 9\,, \tag{A.1}$$
$$U, V, W = 1, 2, \cdots, 7, y, z\,, \tag{A.2}$$
$$I, J = 1, 2, \cdots, 7, 8\,, \tag{A.3}$$
$$m, n, p, q = 1, 2, \cdots, 7\,, \tag{A.4}$$
$$u, v = 8, 9\,. \tag{A.5}$$

The worldvolume, worldsheet and spacesheet indices;

$$\alpha, \beta = 0, 1, 2\,, \tag{A.6}$$
$$a, b = 0, 1\,, \tag{A.7}$$
$$i, j = 1, 2\,. \tag{A.8}$$

[14] Of course the (1,0)-string (F-string) is an effective mode in a weak coupling region $g_b \ll 1$, while the (0,1)-string (D-string) in a strong coupling region $g_b \gg 1$ for $l = 0$.

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The target-space metrics;

$$
\begin{aligned}
G_{MN} &= \text{Target-space transverse metric}\,, && \text{(A.9)}\\
\tilde{G}_{UV} &= \text{Rotated target-space transverse metric}\,. && \text{(A.10)}
\end{aligned}
$$

(Anti-)symmetrization r.w.t. indices;

$$
\begin{aligned}
A_{[\mu}B_{\nu]} &= \frac{1}{2}\left(A_\mu B_\nu - A_\nu B_\mu\right)\,, && \text{(A.11)}\\
A_{[\mu}B_\nu C_{\rho]} &= \frac{1}{3!}(A_\mu B_\nu C_\rho + A_\nu B_\rho C_\mu + A_\rho B_\mu C_\nu \\
&\qquad - A_\mu B_\rho C_\nu - A_\rho B_\nu C_\mu - A_\nu B_\mu C_\rho)\,, && \text{(A.12)}\\
A_{[\mu}B_{|\nu|}C_{\rho]} &= \frac{1}{2}(A_\mu B_\nu C_\rho - A_\rho B_\nu C_\mu)\,, && \text{(A.13)}\\
A_{\{\mu}B_{\nu\}} &= \frac{1}{2}\left(A_\mu B_\nu + A_\nu B_\mu\right)\,, \quad \text{etc.} && \text{(A.14)}
\end{aligned}
$$

B. Background fields

From the KK relation between 11-dimensional supergravity and type IIA supergravity, the transverse metric G_{MN} can be written by

$$
\begin{aligned}
G_{MN} &\equiv e^{-\frac{2}{3}\phi}\begin{pmatrix} g_{IJ}+e^{2\phi}A_IA_J & e^{2\phi}A_I \\ e^{2\phi}A_J & e^{2\phi}\end{pmatrix}\\
&= \begin{pmatrix} \frac{1}{\sqrt{G_{99}}}\,g_{IJ}+\frac{1}{G_{99}}G_{I9}G_{J9} & G_{I9} \\ G_{J9} & G_{99}\end{pmatrix}\\
&= \begin{pmatrix} \frac{1}{\sqrt{G_{99}}}\,g_{mn}+\frac{1}{G_{99}}G_{m9}G_{n9} & \frac{1}{\sqrt{G_{99}}}\,g_{m8}+\frac{1}{G_{99}}G_{m9}G_{89} & G_{m9} \\ \frac{1}{\sqrt{G_{99}}}\,g_{8n}+\frac{1}{G_{99}}G_{89}G_{n9} & \frac{1}{\sqrt{G_{99}}}\,g_{88}+\frac{1}{G_{99}}G_{89}G_{89} & G_{89} \\ G_{n9} & G_{89} & G_{99}\end{pmatrix}, && \text{(B.1)}
\end{aligned}
$$

and the third-rank antisymmetric tensor A_{MNP} is decomposed as

$$
\begin{aligned}
A_{MNP} &= (A_{mnp}, A_{mn9}, A_{mn8}, A_{m89})\\
&= (C_{mnp}, B_{mn}, C_{mn8}, B_{m8})\,. && \text{(B.2)}
\end{aligned}
$$

Those fields are related to those of IIB as,

$$
\begin{aligned}
g_{mn} &= \jmath_{mn} - \frac{\jmath_{8m}\jmath_{8n} - B^{(1)}_{8m}B^{(1)}_{8n}}{\jmath_{88}}\,, && \text{(B.3)}\\
g_{8m} &= \frac{B^{(1)}_{8m}}{\jmath_{88}}\,, && \text{(B.4)}\\
g_{88} &= \frac{1}{\jmath_{88}}\,, && \text{(B.5)}\\
C_{mn8} &= B^{(2)}_{mn} + \frac{2B^{(2)}_{8[m}\jmath_{n]8}}{\jmath_{88}}\,, && \text{(B.6)}\\
C_{mnp} &= D_{8mnp} + \frac{3}{2}\,\epsilon^{ij}B^{(i)}_{8[m}\,B^{(j)}_{np]} + \frac{3}{2}\,\epsilon^{ij}\,\frac{B^{(i)}_{8[m}\,B^{(j)}_{n|8|}\jmath_{p]8}}{\jmath_{88}}\,, && \text{(B.7)}\\
B_{mn} &= B^{(1)}_{mn} + \frac{2B^{(1)}_{8[m}\jmath_{n]8}}{\jmath_{88}}\,, && \text{(B.8)}\\
B_{8m} &= \frac{\jmath_{8m}}{\jmath_{88}}\,, && \text{(B.9)}\\
A_m &= -B^{(2)}_{8m} + lB^{(1)}_{8m}\,, && \text{(B.10)}\\
A_8 &= l\,, && \text{(B.11)}\\
\phi &= \varphi - \frac{1}{2}\ln \jmath_{88}\,. && \text{(B.12)}
\end{aligned}
$$

The modular field of a 2-torus is defined by $\tau \equiv l + i\,e^{-\varphi}$ and can be rewritten as

$$
\tau = \frac{G_{89} + i\sqrt{V_{T^2}}}{G_{99}}\,, \tag{B.13}
$$

where $V_{T^2} \equiv G_{99}G_{88} - (G_{89})^2$.

On the other hand, the 8-9 rotated metric is given by $(U, V = 1, \cdots, 7, y, z)$

$$
\begin{aligned}
\tilde{G}_{UV} &= G_{MN}\,\frac{\partial X^M}{\partial X^U}\,\frac{\partial X^N}{\partial X^V}\\
&= \begin{pmatrix}
\frac{1}{\sqrt{\tilde{G}_{zz}}}\,\tilde{g}_{mn} + \frac{1}{\tilde{G}_{zz}}\tilde{G}_{mz}\tilde{G}_{nz} & \frac{1}{\sqrt{\tilde{G}_{zz}}}\,\tilde{g}_{my} + \frac{1}{\tilde{G}_{zz}}\tilde{G}_{mz}\tilde{G}_{yz} & \tilde{G}_{mz}\\
\frac{1}{\sqrt{\tilde{G}_{zz}}}\,\tilde{g}_{yn} + \frac{1}{\tilde{G}_{zz}}\tilde{G}_{yz}\tilde{G}_{nz} & \frac{1}{\sqrt{\tilde{G}_{zz}}}\,\tilde{g}_{yy} + \frac{1}{\tilde{G}_{zz}}\tilde{G}_{yz}\tilde{G}_{yz} & \tilde{G}_{yz}\\
\tilde{G}_{nz} & \tilde{G}_{yz} & \tilde{G}_{zz}
\end{pmatrix}. && \text{(B.14)}
\end{aligned}
$$

When we choose an SO(2) rotation as in eq. (3.15) the background metric on a 2-torus is given by

$$
\begin{aligned}
\tilde{G}_{zz} &= \hat{q}^2\,G_{88} + 2\hat{p}\hat{q}\,G_{89} + \hat{p}^2\,G_{99}\,, && \text{(B.15)}\\
\tilde{G}_{yy} &= \hat{p}^2\,G_{88} - 2\hat{p}\hat{q}\,G_{89} + \hat{q}^2\,G_{99}\,, && \text{(B.16)}\\
\tilde{G}_{yz} &= \hat{p}\hat{q}\,G_{88} + (\hat{p}^2 - \hat{q}^2)\,G_{89} - \hat{p}\hat{q}\,G_{99}\,. && \text{(B.17)}
\end{aligned}
$$

The background metric on the 2-torus is rewritten by the fields of type IIB superstring theory,

$$\begin{aligned}
G_{99} &= e^{4\varphi/3}\jmath_{88}^{-2/3}\,, &\text{(B.18)}\\
G_{89} &= e^{4\varphi/3}\jmath_{88}^{-2/3}\,l\,, &\text{(B.19)}\\
G_{88} &= e^{4\varphi/3}\jmath_{88}^{-2/3}\,(l^2+e^{-2\varphi})\,, &\text{(B.20)}\\
\tilde{G}_{zz} &= e^{4\varphi/3}\jmath_{88}^{-2/3}\left\{(\hat{p}+\hat{q}l)^2+e^{-2\varphi}\hat{q}^2\right\}\,, &\text{(B.21)}\\
\tilde{G}_{yz} &= e^{4\varphi/3}\jmath_{88}^{-2/3}\left\{(\hat{p}l-\hat{q})(\hat{q}l+\hat{p})+\hat{p}\hat{q}e^{-2\varphi}\right\}\,, &\text{(B.22)}\\
\tilde{G}_{yy} &= e^{4\varphi/3}\jmath_{88}^{-2/3}\left\{(\hat{q}-\hat{p}l)^2+\hat{p}^2e^{-2\varphi}\right\}\,. &\text{(B.23)}
\end{aligned}$$

Note that

$$\sqrt{\frac{\tilde{G}_{zz}}{G_{99}}}=\sqrt{(\hat{p}+\hat{q}l)^2+e^{-2\varphi}\hat{q}^2}\,. \tag{B.24}$$

Furthermore,

$$\tilde{G}_{my}=\frac{1}{\sqrt{\tilde{G}_{zz}}}\tilde{g}_{my}+\frac{1}{\tilde{G}_{zz}}\tilde{G}_{mz}\tilde{G}_{yz}\,. \tag{B.25}$$

and hence the 9-dimensional metric is also rewritten by

$$\tilde{g}_{my}=\frac{1}{\sqrt{\tilde{G}_{zz}}}\left(\tilde{G}_{my}\tilde{G}_{zz}-\tilde{G}_{mz}\tilde{G}_{yz}\right)=\frac{\hat{p}\,B^{(1)}_{8m}+\hat{q}\,B^{(2)}_{8m}}{\jmath_{88}\sqrt{(\hat{p}+\hat{q}l)^2+e^{-2\varphi}\hat{q}^2}}\,. \tag{B.26}$$

Furthermore, we shall calculate $\tilde{g}_{mn},\tilde{g}_{yy}$ as follows. The equation,

$$\tilde{G}_{yy}=\frac{1}{\sqrt{\tilde{G}_{zz}}}\tilde{g}_{yy}+\frac{1}{\tilde{G}_{zz}}\tilde{G}_{yz}\tilde{G}_{yz}\,, \tag{B.27}$$

leads to

$$\tilde{g}_{yy}=\frac{1}{\sqrt{\tilde{G}_{zz}}}\left(\tilde{G}_{yy}\tilde{G}_{zz}-\tilde{G}_{yz}\tilde{G}_{yz}\right)=\frac{1}{\jmath_{88}\sqrt{(\hat{p}+\hat{q}l)^2+e^{-2\varphi}\hat{q}^2}}\,. \tag{B.28}$$

Similarly

$$G_{mn}=\frac{1}{\sqrt{\tilde{G}_{zz}}}\tilde{g}_{mn}+\frac{1}{\tilde{G}_{zz}}\tilde{G}_{mz}\tilde{G}_{nz}\,, \tag{B.29}$$

leads to

$$\begin{aligned}
\tilde{g}_{mn} &= \frac{1}{\sqrt{\tilde{G}_{zz}}}\left(G_{mn}\tilde{G}_{zz}-\tilde{G}_{mz}\tilde{G}_{nz}\right)\\
&= \sqrt{(\hat{p}+\hat{q}l)^2+e^{-2\varphi}\hat{q}^2}\left(\jmath_{mn}-\frac{\jmath_{8m}\jmath_{8n}}{\jmath_{88}}+\frac{(\hat{p}B^{(1)}_{8m}+\hat{q}B^{(2)}_{8m})(\hat{p}B^{(1)}_{8n}+\hat{q}B^{(2)}_{8n})}{\jmath_{88}\left\{(\hat{p}+\hat{q}l)^2+e^{-2\varphi}\hat{q}^2\right\}}\right)\\
&= \sqrt{(\hat{p}+\hat{q}l)^2+e^{-2\varphi}\hat{q}^2}\left(\jmath_{mn}-\frac{\jmath_{8m}\jmath_{8n}}{\jmath_{88}}+\frac{B^{(pq)}_{8m}B^{(pq)}_{8n}}{\jmath_{88}}\right)\,. \qquad \text{(B.30)}
\end{aligned}$$

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